B Homework: QED (due Thu 13th Mar 5pm)

B.1 Feynman diagrams

(a) Using only the QED vertex (4.4), draw the lowest order (in $\alpha_{\rm QED}$) Feynman diagrams for:

(i) $e^- e^+ \to e^- e^+$,

(ii) $e^- \mu^+ \to e^- \mu^+$,

(iii) $\gamma\gamma \to \gamma\gamma$,

(iv) $\gamma p \to e^- e^+ p$ treating the proton p as a point particle

State how many powers of $\alpha_{\rm EM}$ are in the cross-section for each process.

(b) Why can $e^- e^+ \to \mu^- \mu^+$ not proceed via a t-channel diagram?

(c) Given the range R of a massive force carrier is $R = 1/m$, restore $\hbar c$ to numerically estimate R for a pion ($m_\pi \approx 140$ MeV) and W boson ($m_W \approx 80$ GeV).

B.2 Gauge theory

Suppose we rescaled the phase in equation (4.46) to a general charge $Q\beta(x)$ such that $\psi(x) \to \psi'(x) = \psi(x)\mathrm{e}^{\mathrm{i}Q\beta(x)}$. Inserting this into the Dirac equation with an appropriately adjusted covariant derivative and following similar steps to arrive at (4.50), show what the transformation $A^\mu \to A^{\mu\prime}$ must be to restore invariance of the Dirac equation.

B.3 Feynman rules to cross-section

(a) Using the Feynman rules in section 4.4 (or textbook), write down the amplitude of the four Feynman diagrams sketched in figure 25: (i) $e^- e^- \to e^- e^-$, (ii) $e^- e^+ \to \gamma\gamma$, (iii) $e^- \gamma \to e^- \gamma$, (iv) $\gamma\gamma \to e^- e^+$. Draw the diagrams and label the parts that correspond to mathematical expressions for the spinors u, v, polarisation vectors ε_μ, and propagators.

(b) Considering ultra-relativistic $e^- e^+ \to \mu^- \mu^+$ scattering for the non-zero helicity combinations (6.7), sketch the arrows denoting the momentum $\mathbf{p}$ and helicity $\mathbf{s}$ vectors for the ingoing electron and positron, and outgoing muon and anti-muon.

(c) For each of the four possible helicity combinations, sketch graphs of $\mathscr{A}$ vs. $\cos\theta$ based on the amplitude dependence $\propto (1 \pm \cos\theta)^2$ from eq. (6.20).

(d) Starting from the differential cross-section $\mathrm{d}\sigma = \frac{1}{\Phi}|\mathscr{M}|^2 \mathrm{d}n_{\text{2-body}}$ with the incident flux $\Phi = 2s$, the averaged sum over the possible spin states $|\mathscr{M}|^2 = \frac{1}{4}\sum_{\rm spins}|\mathscr{A}_{\rm spins}|^2$ from the lecture notes eq. (6.21), and the 2-body Lorentz invariant phase space $\mathrm{d}n_{\text{2-body}} = \frac{p_f}{E_{\rm CM}}\frac{\mathrm{d}\Omega}{(4\pi)^2}$ from eq. (5.38), show the steps to obtain $\sigma = \frac{4}{3}\frac{\pi\alpha_{\rm EM}^2}{s}$.