C Homework: Strong force (due Thu 10th Apr 5pm)

C.1 Form factors

(a) Qualitatively sketch and describe the form factor for an electron scattering off a point-like and hard-sphere charge distribution.

(b) Using the Born approximation, perform the integration to find the form factor given an exponentially decaying charge distribution of a proton: $\rho(r) = \exp\left[-r^2/(2a^2)\right]$.

C.2 Colour charge and potential

(a) Write a few sentences discussing the evidence for colour charge based on (i) Fermi-Dirac statistics and quark wavefunction arguments of the Ω^- baryon comprising three strange quarks, and (ii) the ratio of $R = \sigma(e^-e^+ \to \text{hadrons})/\sigma(e^-e^+ \to \mu^-\mu^+)$.

(b) Consider the colour potential of equation (10.40) $V(r) = -\frac{4}{3}\alpha_s \frac{\hbar c}{r} + \lambda r$. Determine the critical r_c when the two terms in $V(r)$ are equal. Sketch two quarks being pulled apart as a function of r for $r < r_c, r \sim r_c, r > r_c$. Sketch how at small inter-quark distance $r < r_c$, the attractive force from the gluon field increases. At large inter-quark distance $r > r_c$, sketch how quarks are created from the energy in the gluon field, before combining to hadrons.

C.3 Quarks and gluons

(a) Draw the lowest-order tree-level Feynman diagrams for $u\bar{u} \to u\bar{u}, dg \to dg, gg \to gg, gg \to t\bar{t}$. (b) Draw the Feynman diagram showing electron-proton scattering, treating the proton as a point-like particle and scattering off internal quarks. Label the momenta for the incident and ongoing particles. Write a few sentences discussing the experimental evidence for the existence of quarks and their properties in deep inelastic scattering experiments.

C.4 QCD and running coupling

(a) Discuss the theoretical similarities and differences between the gauge theory of quantum electrodynamics and quantum chromodynamics.

(b) From the fine structure constant of eq. (7.21) $\alpha\left(Q^2\right) = \alpha_0/\left[1 + (\alpha_0/3\pi)\ln\left(\Lambda^2/Q^2\right)\right]$, eliminate the immeasurable bare coupling α_0 and cutoff scale Λ to derive equation (7.22) $\alpha\left(Q^2\right) = \alpha(\mu^2)/\left[1 - [\alpha(\mu^2)/3\pi]\ln\left(Q^2/\mu^2\right)\right]$ in terms of measurable quantities. Hint: use a reference scale m_e by subtraction $\alpha^{-1}(Q^2) - \alpha^{-1}(m_Z^2)$. Sketch $\alpha(Q^2)$ as a function of Q^2.